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A Synthesis of Research on Deaf and Hearing Children's Mathematical Achievement

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Over five decades, researchers have reported that deaf children lag behind their hearing peers on different educational measures. This review aims to synthesize the information on the nature and extent of this delay. A systematic search of the literature comparing deaf and hearing children's performance in mathematics was carried out. Of the 23 relevant articles, 13 employed standardized measures and 10 used un-standardized measures.

The analysis demonstrates that, for children whose level of hearing loss is greater than moderate, there is a delay in mathematics in comparison with hearing children. This delay is noted in all assessments with standardized measures. Three studies (about 30%) that used un-standardized measures reported no delay; in these studies the children did not have to use conventional mathematical signs in order to solve the tasks. This may indicate that deaf children's number representation is not impaired but their learning of conventional mathematical signs is delayed.

An attempt was made to quantify the level of delay, but the findings must be interpreted with caution due to possible confounding in the studies, some related to the sample (inclusion and exclusion criteria) and others related to the lack of controls (comparisons with only chronologically age-matched controls when general intelligence should have been controlled for).

KEYWORDS deaf children's mathematics, comparisons with hearing children, mathematics achievement, standardized measures

Introduction

The aim of this paper is to understand the nature and the extent of deaf children's delay in mathematics in comparison to hearing cohorts.

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In view of the relatively limited number of studies, a best-evidence synthesis approach was adopted. This incorporates the methods of meta-analysis with the detailed analysis of critical issues and study characteristics in order to synthesize research, providing clear and useful conclusions (Slavin, 1986).

Method

A search for relevant publications was carried out using PSYCHO-info, SCOPUS, ERIC, and British Education Index electronic databases using the following descriptors: *mathematics, achievement, problem solving, arithmetic, deaf and hearing impaired*. Studies published since 1965 were included in order to capture as many studies as possible but still reflect current knowledge.

This search identified about 400 relevant articles. These were evaluated using the criteria that only studies that reported results on deaf children's mathematical performance and allowed for a comparison with hearing children were included. Twenty-three articles satisfied this criterion. The conclusions derived from this investigation should be considered with caution because of the small number of studies in the analysis.

Data analysis

Coding procedures

The studies were divided into those that used standardized measures (13) and those that employed un-standardized measures (10). Standardized measures are normed for different ages and are broad in the coverage of mathematical topics; for example, the Performance Indicators for Primary School (PIPS) includes items on problem solving, analysis of graphs, arithmetic, knowledge of time, pattern identification, and counting. Because standardization is based on large sample sizes and norms such as percentiles or standard deviation are provided, these measures can be compared across studies and their effect sizes can be meaningfully averaged.

Un-standardized measures typically assess specific aspects of mathematical achievement (e.g. number representation, knowledge of the counting string, problem solving, or particular aspects of mathematical reasoning) and do not report percentiles or standard deviations based on large samples. Because of the specificity and diversity of the measures, averaging effect sizes is not meaningful.

Studies were further coded by the publication status (place, year, and authors), description of the participants (age, level of hearing loss, sample size, type of school, presence of children with additional needs, and language used at home), and results (tests used and findings).

Table 1 lists the first group of studies, organized by the country where the investigation took place and by the year of publication. Table 2 reports the second group of studies by the age of participants and mathematical abilities tested. No information about children with additional needs is recorded because none of the studies included children with special needs other than deafness.

TABLE 1

STUDIES THAT CONSIDERED STANDARDIZED MEASURES PRESENTING COUNTRY, YEAR, CHILD'S AGE, LEVEL OF HEARING LOSS, SAMPLE SIZE, TYPE OF SCHOOL, PRESENCE OF ADDITIONAL NEEDS, TEST USED, RESULTS, AND EFFECT SIZE

Place	Year	Articles	Age	Level of hearing loss	*N* deaf	*N* hearing	Type of school	Additional needs	Language	Test	Results	Effect size
UK	1965	Wollman	14–16	Mild to profound	1/4 to 1/3 of children of this age	162	Specials schools	nr	nr	Manchester Mechanical Arithmetic Test and a test with arithmetical problems	1 sd below hearing	–1.00
UK	*1970*	*Hine*	*7.8–16.5*	*Mild to moderate*	*104*	*Norms*	*One school for hearing impaired*	*nr*	*nr*	*Schonnel's Essential Mechanical Problem Arithmetic tests*	*10 years old 2 years below; 15 years old 5–4 years below*	*nr*
UK	1983	Wood et al.	15–16	Moderate to profound	414	465	Special schools and hearing units	nr	nr	Vernon and Miller Graded Arithmetic–Mathematics test	3–4 years below hearing	nr
UK	*1998*	*Nunes and Moreno*	*8–11*	*Mild to profound*	*85*	*Norms*	*Special schools and mainstream with units for deaf*	*no*	*BSL/ English*	*NFER-Nelson 7–11*	*2 sd below the mean of hearing children*	*–2.00*
UK	2003	Tymms et al.	4–5	Mild to profound	962	2000	Different schools	more than half	nr	Performance Indicators in Primary Schools	Mild hearing loss no difference with hearing	On entry: –0.52; – 0.62; –0.81; –0.84; End KS1: –0.36; – 0.48; –0.39; –0.55
UK	2006	Thoutenhoofd	5–12	Mild to profound	152	1752	nr	131	nr	National Test of the 5–14 Curriculum	Only 34.2% deaf performed D+ against 60% of hearing	nr
UK	2009	Gottardis	7–9	Mild to moderate	86	5973	Special schools and mainstream with units for deaf	no	nr	WISC arithmetic test, KS1, mathematical reasoning	No difference between deaf and hearing	WISC: –8.8; KS1: 0.15; math reasoning: 6
US	*1986*	*Allen*	*8–18*	*nr*	*in 1974:6158 in 1983:7004*	*nr*	*Special education services*	*nr*	*nr*	*Stanford Achievement Test–6th and 7th ed.*	*1–3 years below hearing*	*1974: −1.21; 1983: −0.15*

Continued

TABLE 1

Place	Year	Articles	Age	Level of hearing loss	*N* deaf	*N* hearing	Type of school	Additional needs	Language	Test	Results	Effect size
US	2000	Traxler	8–18	Mild to profound	971	4808	Children selected by teachers	nr	nr	Stanford Achievement Test–9th ed.	Below basic	Problem solving: –0.4; Procedures: –0.9
US	*2007*	*Qi and Mitchell*	*8–17*	*Mild to profound*	*5 cohorts from 1974 to 2003*	*nr*	*nr*	*nr*	*nr*	*Stanford Achievement Test*	*Below basic*	*nr*
US	*2009*	*Krilzer*	*4–6*	*nr*	*29*	*Norms*	*Recruited from 7 schools across US*	*no*	*ASL/ English*	*Test of Ealry Mathematics Ability (TEMA–3)*	*14.29% Above average, 21.43% 2–6 months below, 25% 7–10 months below, 39.9% 12–22 months below*	*–0.51*
US	*2009*	*Antia et al.*	*6–14*	*Mild to profound*	*197*	*Norms*	*General education classrooms*	*no*	*ASL/ English*	*Stanford Achievement Test–9th ed.*	*Below average*	*–0.19*
Norway	1996	Frostad	7–16	Moderate to profound	200	196	Special schools, special units and local school	29	nr	Computational Test	Below average	–0.50

TABLE 2

STUDIES THAT CONSIDER NON-STANDARDIZED MEASURES PRESENTING COUNTRY, PUBLICATION'S YEAR, LEVEL OF HEARING LOSS, SAMPLE SIZE, TYPES OF SCHOOLS, CHILDREN'S LANGUAGE, TEST USED, RESULTS, EFFECT SIZE AND STANDARD ERROR

Place	Year	Articles	Age	Level of hearing loss	*N* deaf	*N* hearing	Type of school	Language	Test	Results	Effect size	Standard error
UK	2004	Zarfaty et al.	2.5–4.5	Moderate and profound	10	20	specialist nursery school	nr	Number rep resentation	Deaf and hearing smilar performances	Spatial: 0.4; Temporal: 0.11	Spatial: 1.67; Temporal: 1.48
US	2005	Bull et al.	18–28	nr	20	20	Rochester Institute of Technology	BSL	Number rep resentation	Deaf perform on the 15th percentile in comparison to hearing adults	–0.45	0.55
Belgium	2002	Leybaert and Van Cutsem	3–6	Moderate to profound	21	28	Special schools	sign language	Abstract counting	Hearing outperformed deaf	–1.49	0.32
Brasil	2010	Barbosa	5–6		11	33	State schools	sign language	Number representation and abstact counting	Deaf and hearing smilar performances on representation but lower in counting	Representation: 0.08 Counting: –1.58	Representation: 0.37 Counting: 0.42
Italy	2011	Arfe et al.	5.2	Profound	10	99	Mainstream	Italian	Counting task, disit comparison, analogic comparison	Deaf children outperform hearing children	Analogic counting: 0.9 Digit counting: –0.14 Verbal counting: –0.26	0.33
UK	2008	Nunes et al.	5–8.3	Moderate to profound	23	130	Special schools and mainstream with special units	BSL	Inverse between addition and subtraction	Hearing outperformed deaf children	–0.10	0.22
UK	2008	Nunes et al.	6–7	Moderate to profound	28	78	Special schools and mainstream with special units	English	Multiplicative reasoning task	Hearing outperformed deaf children	Simultaneous: –6.4 Successive: –11.21	Simultaneous: 0.49; Successive: 0.79
UK	2009	Nunes et al.	6–7	Moderate to profound	28	77	Supported schools	nr	Additive composition task	Hearing outperformed deaf children	–2.59	0.29
US	1995	Titus	10–16	Mild to profound	21	26	Residential school for the deaf	nr	Fractional number instrument	Hearing outperformed deaf children	Young: –1.01; old: –2.72	Young: 0.31; Old: 0.4
US	2007	Blatto-Vallee et al.	13–20	Severe to profound	149	156	nr	nr	15 mathematical problems adapted from the MPI	Hearing students outperformed deaf children	Middle school: –0.72; High school: –1.70; College: –1.25	Middle school: 0.11; High school: 0.13; College: 0.12

The studies in italics compared deaf children's performance to norms for hearing children; other studies had comparison groups of hearing children. Two studies (underlined) only included deaf children with cochlear implants.

Effect size calculation

Statistical information required for the calculation of the effect size and the standard error of measurements was obtained for 19 studies either directly from the publications or from correspondence with the authors. For most, the effect size calculated was Cohen's *d*, using the weighted standard deviation between groups. In one case (Allen, 1986), the effect size was estimated using information from the figure presented in the article. For two studies (Hine, 1970; Wood et al., 1983), a measure of delay was obtained by considering the gap between the children's chronological and mathematical age.

The standard error was calculated only for the small-scale studies using Hedges and Olkin's (1985) formula.

Results

The results are reported separately for studies with standardized and non-standardized measures. For each group, we consider first the study features, highlighting the population characteristics and the design. Second, the extent of the delay is addressed considering whether all studies report a delay in deaf children's mathematics and analyse the effect sizes.

Study features

Studies with standardized measures

Sample characteristics

A total of 16,362 deaf children were included, but sample sizes varied widely, ranging from twenty-nine to 7004 participants per study [Wollman (1965) and Qi and Mitchell (2007) did not report the exact number of children recruited]. The deaf children were compared with a total of 15,356 hearing children [Hine (1970), Allen (1986), Nunes and Moreno (1998), Qi and Mitchell (2007), Kritzer (2009), and Antia et al. (2009) did not report the exact number of hearing children].

The average age of the children was 12.8 years with a range from 4.5 to 15.5 years; the level of hearing loss covered the whole range of hearing losses (mild to profound). Two studies (Hine, 1970; Gottardis, 2009) considered only children with mild to moderate hearing loss.

The type of school attended by the children was most often special schools or mainstream schools with a unit for deaf children.

Three studies (Frostad, 1996; Tymms et al., 2003; Thoutenhoofd, 2006) recruited children with additional needs.

Only two studies (Hine, 1970; Gottardis, 2009) reported the socio-economic background of the children; they found no difference between deaf and hearing children's socio-economic status.

Study design

Nine studies (Wollman, 1965; Hine, 1970; Wood et al., 1983; Frostad, 1996; Nunes & Moreno, 1998; Tymms et al., 2003; Thoutenhoofd, 2006; Kritzer, 2009; Antia et al., 2009) carried out a survey and four studies (Allen, 1986; Traxler, 2000; Qi & Mitchell, 2007; Gottardis, 2009) were secondary data analyses. In all studies, the criterion used to match deaf and hearing children was chronological age. The use of chronological age as the matching criterion could lead to over-estimating differences between deaf and hearing children, when the differences may be attributed to other factors rather than deafness, such as cognitive development (if the children had additional needs) or educational level (if deaf children's entry into school is delayed).

Studies with non-standardized measures

Sample characteristics

In these studies, 321 deaf children were compared with a total of 667 hearing children. The range in number of participants per study was from 10 to 149. The average age across studies was 9.02 years; the ages ranged from 3.5 to 23 years.

The children recruited had a hearing loss from moderate to profound; they attended special schools or were included in mainstream schools with a special unit for deaf children. In only four studies (Leybaert & Van Cutsem, 2002; Bull et al., 2005; Nunes et al., 2008; Barbosa, 2010) the children used sign language.

Study design

In five studies (Titus, 1995; Zarfaty et al., 2004; Bull et al., 2005; Nunes et al., 2008; Arfè et al., 2011), deaf children were matched to hearing children by chronological age. Leybaert and Van Cutsem (2002) and Blatto-Vallee et al. (2007) matched deaf to hearing children by level of education; Nunes et al. (2008), Nunes et al. (2009a, 2009b), and Gottardis (2009) carried out analyses that controlled for the children's non-verbal intelligence, above and beyond chronological age, when hearing and deaf children were compared.

Extent of the delay

Do all studies report a delay in deaf children's mathematics achievement?

In all but four studies a delay in deaf children's mathematics achievement is reported. The first of two similar studies that did not report a delay was by Zarfaty et al. (2004), which showed a positive effect size (Cohen's $d = 0.25$). They investigated whether deaf children at kindergarten were delayed in number representation tasks that could be accomplished without counting. This is the only study that analysed performance in such a young sample. Zarfaty et al. (2004) hypothesized that deaf children have a preference for processing information that is displayed simultaneously and can be represented using the visuo-spatial sketch pad in working memory (Gathercole et al., 2004), whereas they are at a disadvantage when the information is presented successively and is more easily represented using the phonological loop. Deaf children showed better performance in the number representation task than their hearing peers when the items were displayed simultaneously and did not differ from hearing children when the items were presented successively. The preschool deaf children's performance was at least as advanced as that of hearing children in these non-verbal tasks. It has been suggested by some researchers (e.g. Landerl et al., 2004) that the origins of severe difficulty in mathematics can be found in children's difficulties with early number representations that do not depend on language. Thus, Zarfaty et al.'s (2004) study provides a test of whether this explanation could account for deaf children's underachievement in mathematics. The researchers concluded that deaf children's delay in mathematics achievement cannot be explained by difficulties in their early number representation and should be searched for in their learning and use of conventional systems of signs to solve mathematical tasks. In Brazil, Barbosa (2010) carried out a similar study and replicated the results of Zarfaty et al. with deaf children aged 5–6 years and confirmed that deaf children's number representation ability was as good as that of hearing children when counting was not required.

The third study that does not show a delay was by Arfè et al. (2011), who explored how preschoolers with cochlear implants process numerical comparisons from two different inputs: non-verbal (analogical) and verbal (symbolic). They compared the counting abilities of ten children with cochlear implants with that of ninety-nine hearing children on three tasks. In the first task, the children were asked to count aloud from memory from one to twenty. In the second task, the children had to choose the larger number between two digits named by the examiner and presented visually on cards. The last task has the same structure of the previous one but the quantities were presented with dots instead of Arabic numbers. Arfè et al. (2011) found that, only in this last task, deaf children differ from hearing children, outperforming hearing children. These results confirm further the findings reported by Zarfaty et al. (2004) and Barbosa (2010).

The fourth study that does not show a delay was by Gottardis (2009), who compared the mathematical performance of children aged 7–9 years with mild or moderate hearing loss to hearing children's performance. Gottardis (2009) worked with a sub-sample from the Avon Longitudinal Study of Parents and Children. Her analysis included 5973 hearing children and 86 children with a hearing loss ranging from mild to moderate. Gottardis (2009) reported that deaf children did not differ in mathematical abilities from hearing children of the same age in three mathematical measures: Key Stage 1 assessments, which are designed by the government in England and administered by teachers in school; a mathematical reasoning task designed by Nunes and Bryant (see Nunes et al., 2009a, 2009b); and the WISC arithmetic subtest (Wechsler, 1992). Gottardis (2009) suggested that these results must be interpreted with caution for two reasons: first, all participants were attending mainstream schools, which means that they may not have been a representative sample of hearing impaired children, because the sample might comprise only children who function well enough in the hearing world. Second, there was a loss of participants in the mathematics assessments, which might indicate that teachers did not included all hearing impaired children in the assessments.

In summary, three studies observed that, in the absence of the need to count, deaf children were not in disadvantage in comparison to hearing children. The sample in these studies was rather small so replication with larger samples is desirable. The fourth study examined the mathematical achievement of primary school children with mild or moderate loss and did not find a delay in comparison to hearing children. However, Gottardis (2009) pointed out the need to be cautious in making generalizations from her sample. In spite of these caveats, one could tentatively conclude that young deaf children do not have an inherent delay in number representation and that children with mild loss may not show a significant delay in comparison to hearing children even in tasks that require the use of counting or arithmetic knowledge.

It would be unwise to extend this conclusion to children with moderate loss, in view of the study by Hine (1970), described in the subsequent section, which found a considerable delay.

What is the extent of the delay with hearing losses beyond mild?

This question is addressed separately according to the two groups of studies. For the studies that used standardized measures, a quantitative overview is presented followed by a more analytical approach in which a comparison across studies investigates the possible impact of five characteristics that may affect the extent of the delay: (1) level of hearing loss; (2) educational provision; (3) inclusion of children with additional needs; (4) age at which the comparison took place; and (5) presence of cochlear implant.

In the studies that employed un-standardized measures, the critical analysis focuses on the measures used.

Studies with standardized measures

Quantitative overview

Effect sizes were obtained from seven studies. Tymms et al. (2003), Nunes and Moreno (1998), and Wollman (1965) reported the effect size and it was possible to calculate the effect size from the results obtained by Kritzer (2009), Traxler (2000), Antia et al. (2009), and Frostad (1996), who reported all the necessary information. The Cohen's *d* effect sizes for these studies were −0.51, −2, −1, −0.51, −0.4, −0.19, and −0.5, respectively.

In the remaining studies, the extent of deaf children's delay is noted in terms of the discrepancy between chronological and mathematical age. Three studies (Hine, 1970; Wood et al., 1983; Allen, 1986) are included under this analysis.

Hine (1970) described the mathematical attainment of 104 deaf children aged 7.8–16.5 years with an average hearing loss of 66.1 dB, which corresponds to a moderate loss. Using the Schonell's Essential Mechanical Problem Arithmetic tests, he observed that deaf children fell behind their hearing peers at all age levels. The 8-year-olds had an average arithmetic attainment of 7.5 years, the 10-year-olds obtained 8.5 years, and the 15-year-olds reached 10.5 years in mechanical arithmetic and 11 years in problem arithmetic.

A similar pattern was observed by Allen (1986). He compared the mathematical achievement of the standardization sample for the seventh edition of the Stanford Achievement Test (SAT) in 1983 with that of the standardization sample for the sixth edition of the test in 1974. The first cohort had 6158 deaf children and the second sample considered 7004. He reported that deaf children lagged behind their hearing peers from 1 to 3 years in mathematical achievement. This delay increased over time and levelled off at 3 years of delay when the students were in the age range 16–18 years.

In the UK, Wood et al. (1983) analysed the mathematical achievement of 414 school leavers aged 15–16 years and compared their performance on the Vernon and Miller Graded Mathematics Test with that of 465 hearing students of the same age. Deaf students had a hearing loss ranging from mild to profound. The mathematical age of hearing students was about 15.5 years whereas the mathematical age of deaf students was 12.3 years, corresponding to a delay of more than 2 years.

For two studies (Thoutenhoofd, 2006; Qi & Mitchell, 2007) the calculation of the effect size was not possible due to lack of information in the publications. Thoutenhoofd's study (2006) included only children with cochlear implants and compared their mathematical performance with that of children with different levels of hearing loss and with hearing children's performance. He observed that 41.9% of deaf children performed at level D (highest level in primary school) in comparison to the 74.4% of the hearing children. Qi and Mitchell (2007) analysed five cohorts of data on students who had taken standardized but different editions of the SAT between 1974 and 2003. They observed that deaf children were behind their hearing peers in all the five cohorts, but did not report any figures that could be used to quantify the delay.

In conclusion, all these studies converge in demonstrating that deaf children lag behind their hearing peers in mathematics. For those studies that presented an effect size, a combined Cohen's *d* effect size of −0.7 was obtained weighting by the sample size. This is considered by Cohen (1992) between a moderate and large effect size.

Factors that might affect the results of the comparison are as follows.

1. *Level of hearing loss*

Table 3 summarizes the studies by the level of the participants' hearing loss, highlighting the effect sizes and the correlation between degree of hearing loss and mathematics.

Contradictory results are noted across studies when the relationship between level of hearing loss and children's mathematical performance is investigated: some studies showed no correlation (Wollman, 1965; Tymms et al., 2003), others indicated low correlations (Wood et al., 1983; Nunes & Moreno, 1998), even if significant. These differences may result from the levels of hearing loss of the children included in the study. If a study includes a narrower range of hearing loss, it may fail to reveal a correlation because the levels of loss are not sufficiently distinct. This may be the case of the three studies (Hine, 1970; Tymms et al., 2003; Gottardis, 2009) that analysed the performance of children with mild and moderate losses. Hine (1970) noted a delay of 2 years in deaf children's mathematical age but did not report the correlation between level of hearing loss and mathematical achievement.

Wood et al. (1983) and Frostad (1996) included children with losses from moderate to profound, and thus had sufficient variation to observe a correlation, if it does exist. Both studies report a delay in deaf children's achievement (see Table 3) and Wood et al. (1983) reported a correlation of –0.13, which was significant due to the large number of participants, but small. Frostad (1996) did not consider this issue.

All the other studies considered the whole range of hearing loss. Those that investigated whether there was a correlation between degree of hearing loss and mathematics achievement reported no significant correlation, but in some studies the correlation was not reported (see Table 3).

Tymms et al. (2003) published the only study that reported effect sizes in the comparison between deaf and hearing children by level of hearing loss. They found that,

TABLE 3

ARTICLES ORGANIZED BY LEVEL OF HEARING LOSS

Level of hearing loss	Article	Correlation	Effect size
Mild and moderate	Hine (1970)	nr	2 years delay
	Gottardis (2009)	–0.15	0.15
	Tymms et al. (2003)	–0.15	Mild: –0.36 Moderate: –0.48
Moderate to profound	Wood et al. (1983)	–0.13*	3–4 years delay
	Frostad (1996)	nr	–0.50
Mild to profound	Wollman (1965)	Non-significant	–1
	Nunes and Moreno (1998)	–0.18	–2
	Traxler (2000)	nr	–0.65
	Antia et al. (2009)	–0.06	–0.19
	Allen (1986)	Non-significant	–0.68
	Tymms et al. (2003)	–0.15	Severe: –0.39 Profound: –0.55

*Significant result; nr: not reported.

for children with mild hearing loss, the effect size was small (–0.36) whereas for children with profound hearing loss the effect size was large (–0.55).

These results support the hypothesis that there is a relationship between level of hearing loss and mathematics achievement when the whole range of losses is considered, but the correlation is low. Therefore, the severity of deaf children's mathematical delay is to some extent moderated by the level of hearing loss.

2. *Educational provision*

Wood et al. (1983) carried out the only study that included sufficient variation in educational provision to allow for an analysis of its effect on deaf children's mathematical achievement. They considered special schools, mainstream schools with a hearing impaired unit, and children integrated in mainstream schools. They reported a low correlation between children's performance and educational provision. However, when level of hearing loss was considered, educational provision did not account for any more variance. They argued for the need to take educational provision into account when the children's mathematical performance is analysed but the confounding between hearing loss and educational provision does not allow for unambiguous conclusions.

In view of the present policy of inclusion in the UK, an analysis of the relation between educational provision and achievement is urgently needed. However, one should bear in mind that educational provision is likely to be confounded with other factors, such as level of hearing loss, use of a signed language, use of a cochlear implant, and the presence of additional special educational needs. Studies with larger samples reporting these details would clarify the significance of educational provision for deaf children's mathematical achievement.

3. *The presence of additional special educational needs*

Table 4 reports the studies that explicitly stated whether children with additional special education needs were included.

Tymms et al.'s study (2003) was the only one that stated clearly that, for hearing losses ranging from mild to severe, the presence of an additional special educational need was significantly and negatively associated with performance in mathematics.

The comparison of effect sizes observed in studies that included children with additional needs and studies that did not might shed some light on the impact of

TABLE 4

STUDIES WITH OR WITHOUT CHILDREN WITH ADDITIONAL NEEDS AND THEIR EFFECT SIZES

Additional needs		No additional needs	
Articles	Effect sizes	Articles	Effect sizes
Tymms et al. (2003)	–0.51	Nunes and Moreno (1998)	–2
Thoutenhoofd (2006)	nr	Kritzer (2009)	–0.51
Frostad (1996)	–0.50	Antia et al. (2009)	–0.19

additional special educational needs on deaf children's mathematical achievement. Of the studies that excluded children with additional needs, one produced a large effect size (−2), one had a medium effect size (−0.5), one had a small effect size (−0.19), so a range of results was observed. It is important to note that in Antia et al.'s (2009) study, which reported the small effect size of −0.19, all the children attended mainstream schools with a unit for hearing impaired children, whereas the other studies included children from different educational provisions. The results reported by Wood et al. (1983) indicate that exclusion of children with additional special educational needs and attending main stream schools may be confounded in the samples participating in different studies. It is also important to note that the criterion for recognizing special educational needs could vary across studies, as in some studies exclusion is only applied if the children have received a statement and in others a teacher's report may be sufficient. It would therefore be important for researchers to indicate how this exclusion criterion was applied.

The presence of a very large effect size in the study by Nunes and Moreno (1998) could be due to the age of the participants and the measure used in the assessment. They evaluated 7- to 11-year-old's mathematical achievement using the NFER-Nelson Graded Arithmetic Test, which relies less on verbal instructions than the other assessments by presenting most of the material visually. The significance of age as a moderator of deaf children's delay in mathematical tests is not yet clear but must not be ignored.

From these results, it is not clear whether the presence of additional needs could partially account for the delay that deaf children have in mathematics and no firm conclusions can be reached. However, one can conclude that studies that exclude children with additional special educational needs do show a delay in deaf children's mathematical achievement.

4. *Age of the participants*

Table 5 summarizes the studies according to the age at which the comparison took place.

It is possible to observe that, even prior to the onset of formal schooling, as in the study of Kritzer (2009) or Tymms et al. (2003), deaf children lag behind their hearing peers in mathematics. When older children were assessed, a larger delay was observed, with the exception of the study by Antia et al. (2009). This is clearly demonstrated by Hine (1970), Allen (1986), and Traxler (2000), who analysed the mathematical performance of deaf children aged from 8 to 18 years. Deaf children in the first years of school presented a delay of a year while in the last years of school the delay increased to approximately 3 years. This trend is further confirmed by Wood et al. (1983) and Wollman (1965) who considered children aged 14–16 years and reported a delay of 1 SD or 3–4 years.

The only study that presents a very small effect size (−0.19) is that of Antia et al. (2009). As previously pointed out, this could be due to the inclusion of only one

TABLE 5

ARTICLES ORGANIZED BY THE AGE OF THE PARTICIPANTS REPORTING THE MEASURES AND THE EFFECT SIZE

Age	Article	Measure	Effect size
4–5	Tymms et al. (2003)	PIPS	−0.51
4–6	Kritzer (2009)	TEMA-3	−0.51
5–12	Thoutenhoofd (2006)	National Test of 5–14 curriculum	nr
7–9	Gottardis (2009)	WISC arithmetic; KS1; mathematical reasoning	0.15
8–11	Nunes and Moreno (1998)	NFER-Nelson Graded Arithmetic-Mathematics test	−2
6–14	Antia et al. (2009)	SAT	−0.19
7–16	Frostad (1996)	Computational test	−0.50
7.8–16.5	Hine (1970)	Schonnel's Essential Mechanical Problem Arithmetic tests	2 years delay
8–18	Allen (1986)	SAT	−0.68
8–18	Traxler (2000)	SAT	−0.65
8–17	Qi and Mitchell (2007)	SAT	nr
14–16	Wollman (1965)	Manchester Mechanical Arithmetic test and a test with arithmetic problems	−1
15–16	Wood et al. (1983)	Vernon and Miller Graded Arithmetic-Mathematics test	3–4 years delay

educational provision (mainstream school) in comparison to the wider range presented in the other studies.

5. *Children with cochlear implants*

The impact of cochlear implants on children's educational achievement is still under investigation. Most studies have focused on language and literacy acquisition. In these domains, there is some evidence of positive effects of cochlear implants, although the issue is still under debate (Geers, 2004; Mayer, 2007; Archbold et al, 2008; Harris & Terlektsi, 2010).

Only one study so far, by Thoutenhoofd (2006), included an analysis of mathematical attainment. Children with cochlear implants were behind their hearing peers in different educational measures; their performance in mathematics was comparable to that of children with moderate hearing loss. This study did not report the statistical significance of the differences and the information was insufficient to calculate the effect size. Thus, it is not possible to reach any conclusions about the impact of cochlear implants from this study.

Studies with non-standardized measures

Studies using non-standardized measures are designed to investigate specific aspects of mathematics. Thus they differ greatly in aims and design, and the variation in effect sizes is considerable (from 0.11 to −11.21). The variation in measures renders the calculation of an overall effect size meaningless, as it would be improper to amalgamate measures that may bear no relationship to one another. Therefore, the analysis of these studies focuses on the nature of the measures and attempts to seek generalizations when possible.

We grouped the studies into: (1) those that used tasks requiring no or very little counting and (2) those that employed tasks that rely on the learning and use of conventional systems of signs. This classification was used because counting is a cultural system that has to be learned and lack of exposure to appropriate learning situations might be one explanation for deaf children's delay in mathematics.

1. *Studies with tasks that do not require counting*

Four studies investigated deaf children's number representation when no counting was required, covering three different age groups. Zarfaty et al. (2004) and Barbosa (2010) examined number reproduction in kindergarten and first year of school, and Bull et al. (2005) assessed college students. In these studies, participants were able to represent the numbers visually because the range of stimuli was within the limits that can be apprehended without counting. In a fourth study, Arfè et al. (2011) analysed the ability to make numerical comparison when the numerical information was presented through dots (which they refer to as analogically). The deaf children had cochlear implants and the age range was 4–5 years. Arfè et al. (2011) observed that the deaf children performed better than the hearing children in this task. Thus, in all four studies, the deaf participants' number skill was at least as good as that of their hearing peers. This convergence in result leads to a tentative conclusion that deaf students' difficulties in mathematics cannot be accounted for by their basic, non-verbal number representation ability.

One further study that required no counting investigated the understanding of the inverse relation between addition and subtraction (Nunes et al., 2008). The children were shown a row of Unifix bricks joined together and asked to count the bricks; the researcher could help the child with counting if necessary. After the experimenter partially hid the row leaving each end of the row exposed, a number of bricks was added to one side of the row and subtracted from the other side. The children were asked how many bricks remained. Although the tasks used numbers, counting itself had no role in the solution of problem. Nunes et al. (2008) found that the deaf children's performance was weaker than their hearing peers, but the effect size was quite small (–0.1). These results further support the conclusion that, when limited counting is required and visual support is given, deaf children's mathematical abilities appear quite similar to those of their hearing peers.

2. *Studies that rely on counting*

Table 6 summarizes the studies that used tasks that rely more on counting.

In all these studies, hearing children outperformed deaf children showing a large delay, ranging from –0.72 to –11.21. These results clearly converge in demonstrating a delay in deaf children's learning of mathematical abilities that require the use of conventional signs.

TABLE 6

ARTICLES REPORTING THE AGE, THE MEASURES AND THE EFFECT SIZES

Articles	Age	Measure	Effect size
Leybaert & Van Cutsem (2002)	3–6	Abstract counting: counting as highly as possible	–1.49
Barbosa (2010)	5–6	Abstract counting: how many items were in the sets after being removed	–1.58
Nunes et al. (2008)	6–7	Multiplicative reasoning: solving problems with either simultaneous or successive presentation	Simultaneous: –6.4 Successive: –11.21
Nunes et al. (2009a, 2009b)	6–7	Shop task: buying and selling	–2.59
Titus (1995)	10–16	Fraction: determine order and equivalence of two fractions numbers in pairs	Young: –1.01 Older: –2.72
Blatto-Vallee et al. (2007)	13–20	Problem solving: 15 problems varying in lengths and difficulty of the language	Middle school: –0.72 High school: –1.70 College: –1.25

Discussion and conclusions

The aim of this synthesis was to investigate whether deaf children lag behind their hearing peers in mathematics. Studies conducted since 1965 reporting deaf children's mathematical abilities in comparison to hearing children's were analysed to investigate the extent and nature of this delay.

The effect sizes demonstrated that deaf children lag behind their hearing peers in mathematics with a combined effect size of –0.7 but many factors were identified which moderate the extent of the delay.

First, the degree of hearing loss is significantly but only mildly correlated with mathematical performance. Mild hearing loss is associated with smaller delays in mathematical achievement in comparison to more severe hearing losses. This was observed when children with additional special educational needs were excluded. Thus, level of hearing loss should be taken into account when the delay in deaf children's mathematics is evaluated.

Second, the higher the level of hearing loss is, the more likely the children are to have additional special educational needs. Due to the low number of studies that reported including children with additional needs in this review, it is unclear whether the presence of additional special needs also moderates the delay in mathematics achievement and further research is necessary.

Third, age seems to be a significant moderator of deaf children's mathematical achievement. Younger children, whose numerical abilities are measured nonverbally, perform as well as hearing children in number representation and comparison tasks. Deaf children's delay starts to appear when the measures place demands on the use of cultural conventions for task solution. The acquisition of the counting string proceeds at a slower pace for deaf than hearing children's. However, when level of counting is controlled for, their ability to use the counting string is comparable to that of hearing children (Leybaert & Van Cutsem, 2002). Although children's knowledge of the counting string *per se* is not a significant predictor of their mathematical ability at a later time, it is unlikely that teachers and parents will engage them in problem solving that requires counting if they do not know how to count beyond very small numbers. Even at school entry deaf children's informal mathematical knowledge is behind that of hearing children when problem-solving tasks are used (Nunes et al., 2008, 2009a, 2009b; Kritzer, 2009). As children start to learn mathematics in school, they may approach hearing children's performance in some standardized tests of arithmetic, but this relative progress seems to be short lived. Deaf children's performance in standardized mathematical tests, which requires more than arithmetic, develops at a slower pace than hearing children's, and the gap between the groups seems to increase until about age 16, when it levels off.

Finally, it is possible that external influences, such as the type of educational provision or the type of hearing device (hearing aids or cochlear implants), may also moderate deaf children's mathematical achievement, but the evidence so far is scarce and ambiguous.

This analysis also identified design issues that need to be addressed in the future. The first concerns whether it is sufficient to match deaf and hearing children by chronological age, thereby assuming that there are no general cognitive differences between these populations and that they would not affect comparisons between samples of hearing and deaf children. Braden (1992) reviewed studies comparing the cognitive abilities of deaf and hearing adults in non-verbal measures of intelligence, and concluded that deafness *per se* does not produce intellectual disadvantage. However, the causes of deafness continue to be unidentified for such a large proportion of deaf children that it is difficult to know whether deafness is the only difference between the samples in such comparisons. We consider a more cautious approach to include controls for non-verbal intelligence when mathematical performance is analysed. In hearing children, previous studies have demonstrated that non-verbal intelligence predicts mathematical achievement (Nunes et al., 2007) and, therefore, controlling for this factor in the investigation of deaf and hearing children's mathematical abilities can provide more accurate estimates of deaf children's delay. The use of non-verbal intelligence as a control could reduce the amount of variance in mathematical achievement specifically related to deafness, but the estimate of effect sizes using this control should be considered in conjunction with the effect sizes obtained when this control is not included, because the latter would provide a more realistic approach to the variation in achievement observed in the classroom.

A second issue is the assumption that the use of non-verbal tasks is a fair assessment of deaf children's abilities. Remine et al. (2007) have found that deaf children's performance on non-verbal measures of intelligence is strongly related to their spoken language skills, which are presumably involved in the understanding of instructions. The role of language in mathematical tasks is noted in the previous results, which show that deaf children perform as well as hearing children in non-verbal number representation tasks, although they underperform in similar tasks when counting is required. A greater awareness and systematic analysis of the linguistic demands of tasks would contribute to a better understanding of deaf children's mathematical achievement and potential for learning.

The third aspect is the selection of the participants. Only a minority of studies reports how the deaf children were selected. An example of the difficulty of not knowing how the sample was selected comes from a study that does report this explicitly. Traxler (2000) clearly stated that the deaf children were selected by their teachers. Through this selection, it is highly possible that the participants were only those who the teacher thought were capable of managing academic assessments. This could lead to an overly optimistic picture of deaf children's abilities. Therefore, future research should be more transparent in this respect.

Finally, it should be pointed out that this review could not address the question of whether the use of a signed or oral language as a medium of education or in the home has a moderator effect on deaf children's mathematical achievement. Nunes and Moreno (1998) investigated whether the use of British Sign Language (BSL) is a good predictor of deaf children's mathematical performance and reported no association between use of BSL at home and mathematics. They argued that the belief that the use of BSL has protective effects on children's learning cannot be accepted without further investigation. Four studies used in this analysis (Leybaert & Van Cutsem, 2002; Bull et al., 2005; Nunes et al., 2008; Barbosa, 2010) included children who use sign language at home but did not address this issue. However, research with standardized measures analysing the use of signed language is so scarce that it is difficult to draw conclusions. Future research should consider this aspect in order to evaluate more clearly whether there is an effect of the home language, the medium of instruction, and the language used in testing on children's mathematics performance.

In conclusion, this review demonstrated that deaf children do show a delay in mathematics in comparison with hearing children, but that this delay does not appear when non-verbal number representation tasks are used. A second contribution was to identify issues for consideration in the design of future research and possible moderators of deaf children's mathematical learning. We see as a major educational implication from these results the incentive to researchers and teachers to attempt to find new ways of giving deaf children access to mathematical tasks by engaging them in tasks that rely on visual resources, with an awareness of the importance of coordinating these with the learning of conventional systems of signs.

Conflict of interest

There are no conflicts of interest.

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