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*Empirical Article*

# The Math Gap: A Description of the Mathematics Performance of Preschool-aged Deaf/Hard-of-Hearing Children

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Over decades and across grade levels, deaf/hard-of-hearing (d/hh) student performance in mathematics has shown a gap in achievement. It is unclear, however, exactly when this gap begins to emerge and in what areas. This study describes preschool d/hh children's knowledge of early mathematics concepts. Both standardized and nonstandardized measures were used to assess understanding in number, geometry, measurement, problem solving, and patterns, reasoning and algebra. Results present strong evidence that d/hh students' difficulty in mathematics may begin prior to the start of formal schooling. Findings also show areas of strength (geometry) and weakness (problem solving and measurement) for these children. Evidence of poor foundational performance may relate to later academic achievement.

Consistently, over decades and across grade levels, deaf/hard-of-hearing (d/hh) students in various countries have scored poorly on mathematics assessments (Leybaert & Van Cutsem, 2002; Mitchell, 2008; Nunes & Moreno, 1998; Pagliaro, Foisack, & Kelly, 2010; Swanwick, Oddy, & Roper, 2005; Traxler, 2000; Wood, Wood, Griffiths, & Howarth, 1986), including tasks involving computation and reasoning (Allen, 1995), logical thinking (Marschark & Everhart, 1999), and problem solving (Ansell & Pagliaro, 2006). Recent data show the vast majority of d/hh students to be significantly below grade level and behind their hearing counterparts in mathematics, exiting high school with approximately a 5th/6th grade level of achievement (Mitchell, 2008; Traxler, 2000). This "gap" in performance is evident in the earlier grades as well, with d/hh students at the 8th, 5th, and even preschool levels already short of expectations (Kritzer, 2009; Traxler, 2000). Although such findings have changed little over time, as indicated by research dating as far back as 1965 (Wollman, 1965), the "math gap" has not been studied in depth, leaving the field with questions as to the origin of the gap and possible implications for future academics and professional success. The current study begins to address these issues. A true understanding of the disparity in performance, however, must begin with an awareness of mathematics development; that is, an understanding of how typical, young children develop foundational understanding of mathematics and the ways in which d/hh children may differ.

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## Early Mathematics Learning

In general, young children's earliest mathematical learning experiences are informal in nature and take place via everyday life events that require them to count, build, share, and group, and that incorporate opportunities for the use of mathematical language and problem solving (Cross, Woods, & Schweingruber, 2009). For example, parents may remark on the number of fingers or toes the child has while bathing or focus the child's attention to a specifically shaped, favorite toy or even refer to the child as the "big brother." Thus, mathematics concepts and skills are "learned" as children make sense of the world around them.

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In a joint position statement on "promoting good beginnings" written by the National Council of Teachers of Mathematics (NCTM) in conjunction with the National Association for the Education of Young Children (NAEYC; NAEYC & NCTM, 2002), the value of early mathematical experiences is reflected in several recommendations that support young children's use of mathematics. These include the building of new concepts on existing mathematical knowledge and experience; the interaction with mathematical ideas; and the use of mathematics during daily activities. Although it is recognized in this position statement that children's understanding of mathematics concepts early in development may be intuitive, a lack of explicit awareness of mathematics concepts may make it difficult for young children to make use of their prior knowledge and to form essential connections when encountering formal mathematics in school. Thus, children between 3 and 6 years of age need to learn how to "mathematize" their environment, that is, to understand mathematically what intuitively makes sense to them (Joint Position Statement of the NAEYC and the NCTM, 2002).

## Mathematics Development

The fields of mathematics education and developmental psychology show that children begin to develop mathematics concepts that are quite complex at a young age (Geary, 1994; Ginsburg & Seo, 2009; Cross et al., 2009; Sarama & Clements, 2009; Gelman & Gallistel, 1978). Infants have been found to demonstrate an implicit awareness of quantity. During the preschool years, formal number knowledge begins to be mapped onto this implicit awareness of quantity as number words and symbols are learned. As young children begin to interact more with their environment, geometry concepts are learned including shapes, locations, and language to describe spatial relationships. Measurement concepts are also beginning to develop and may serve to connect the young child's knowledge and understanding of geometry and number. This understanding of measurement may begin with a perceptual awareness of size and become more refined as numbers are mapped onto this early awareness (Cross et al., 2009).

During these years of early mathematics development, general cognitive skills are also developing. Young children increase their attention spans, learn to stay on task, hold more information in their minds, and acquire the ability to shift between tasks (Cross et al., 2009). Each of these cognitive skills is useful and necessary in mathematics learning.

Several trajectories map the course of early mathematics development (such as Clements & Sarama, 2009; National Council of Teachers of Mathematics (NCTM), 2000; http://www.pbs.org/parents/childdevelopmenttracker/; Sarama & Clements, 2009). Although these sources vary somewhat, they provide a general course of development for the typical child. We provide a synthesis of these trajectories in Appendix A, detailing the development of concepts in number, geometry and spatial sense, measurement, operations and problem solving, and patterns, reasoning, and algebra across approximate ages 2–6 years. In looking at the Table, one can follow the mathematical "growth" of a child through various subconcepts within each larger concept (such as rote counting, counting objects, comparing amounts, etc. for number). For example, in number, a child progresses from counting five objects (~3-year-old level) to counting up to 10 objects (~4-year-old level) to counting up to 20 objects (~5-year-old level). The subconcepts may develop at the same time and may or may not be contingent on one another. Thus, for number, the child may be able to rote count to 10 but may not yet be able to count 10 objects (both at the 4-year-old level, but in different subconcepts). This is similar to a child being able to recite the alphabet by memory or in a song, but not being able to identify the letters in words or to use them to spell. However, the child will not be able to count objects to 10 if he/she cannot first count objects to five (different levels within one subconcept).

Whereas understanding of number is most critical (at least for hearing children) (Gersten, et. al., 2012; Cross et al., 2009), young children's mathematics performance in all concept areas can be used to predict levels of academic achievement well into high school and in content areas beyond mathematics (e.g., reading and science), as well as to identify those who may be at risk for learning disabilities in mathematics (Gersten, et al., 2012). Given the research with d/hh children that shows poor mathematics achievement with this population, and the National Research Council (Cross et al. 2009) recommendations for research with underrepresented

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groups including English Language Learners, it is critical that the nature of the "math gap" that exists between hearing and d/hh children be uncovered and detailed so that specific areas of weakness can be addressed at an early age prior to having an impact on later learning.

## Mathematics Research with Young D/HH Children

Although a relatively small number of studies have investigated the performance of d/hh students on specific areas of mathematics (Pagliaro, 2010), those that do tend to target the school-age population. The few studies known to investigate younger (preschool) d/hh children show mixed findings. Secada (1984) and Leybaert and Van Cutsem (2002) found that d/hh children in the United States and Belgium, respectively, ages 3–7 years and exposed to sign language, demonstrated an age-related lag of approximately two years in their knowledge of the counting string (i.e., rote counting)—perhaps due to limited related opportunities and experiences. These studies, however, also indicated that these signing children had as good or better skills in other subconcepts of number such as object counting and providing the number after a given prompt, especially with lower numbers. The authors attribute this performance to the facility of counting up in signed languages and the tendency of signing d/hh children to track objects with number signs, providing thus a one-to-one correspondence between object and sign.

Zarfaty, Nunes, & Bryant (2004) found that young, oral, d/hh children (ages 3–4 years) in Great Britain could represent number (i.e., duplicate the number of items given in a stimulus) at least as well as their hearing peers when items were presented spatially (i.e., all items presented together in a spatial array) or temporally (i.e., items presented one at a time in sequence), however, the authors concede that their study did not provide complete information related to number knowledge and called for further research in this area.

In a more recent study done in the United States by Kritzer (2009), 28 d/hh children (signing and oral) between 4 and 6 years of age were tested for their mathematics knowledge and skills using the Test of Early Mathematics Ability (TEMA-3). None achieved scores higher than "average" based on normative ranking with the majority receiving scores substantially below average. Eleven children (almost 40%) earned scores that indicated performance of a year or more behind hearing peers. In a secondary qualitative analysis of the videotaped performance of the participants completing the TEMA, specific areas of weakness within the concept of number were noted including story (word) problems, skip counting (counting by twos, threes, etc), number comparison, reading/writing of two to three digit numbers, and addition/subtraction facts.

Less research has been conducted on d/hh students' achievement on mathematics concepts other than those associated with number, such as geometry, measurement, and patterns and categorization. A separate analysis of data from a subsample of the Kritzer study (referred to above) explored the differing mathematical performances of young d/hh children on a categorization task using paper cut-outs that varied in shape and color and buttons that varied in shape, color, and texture. Those children who received relatively high scores on the TEMA-3 demonstrated a more sophisticated awareness of geometric shapes, sorting and labeling shapes such as squares, rectangles, and triangles, whereas children with relatively low mathematical scores were able to label only nongeometrical shapes such as stars and hearts, which tend to vary less in their appearance and thus require limited understanding of the attributes or properties of the shape (Kritzer, 2012). Despite recommendations by the National Research Council in 2009 (Cross et al., 2009) to focus early instruction and research on "geometry, spatial thinking, and measurement" in addition to number, no research has systematically investigated early performance in foundational mathematics concepts outside the area of number with d/hh students. Thus, two research questions drive the present study:

1. To what extent do young d/hh children demonstrate age appropriate understanding of early mathematics concepts in number (i.e., numbering, number comparisons, calculation, concepts, numeral literacy, number facts) as measured by the TEMA-3?
2. In what concept areas (i.e., number and operations; geometry and spatial sense; measurement; and patterns, logic, and algebra) do young d/hh children show strength and weakness?

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## Methodology

### Sample

Twenty children with hearing loss between 3 and 5 years of age from a centralized geographic area within the northeastern part of the United States participated in this study. This area was selected because of its large number of schools/programs for children with hearing loss and its diversity of ethnic groups and socioeconomic status.

All children in the study were participants in the Building Math Readiness in Young Deaf/Hard-of-Hearing Children: Parents as Partners (MRPP) program. The current analysis constitutes the preintervention mathematics achievement assessment of the children, that is, prior to their beginning the intervention. (Data regarding the MRPP intervention program and a description of the intervention is reported elsewhere, Kritzer & Pagliaro, 2012, and will not be discussed in this manuscript.) Families were recruited for the study through various means including advertisements in related parent organizations, various schools/programs for d/hh children, and state commissions, as well as "word of mouth" and personal contacts. No specific criteria beyond age and hearing loss were imposed and children were accepted on a first-come-first-serve basis. All children participating in the study were currently receiving some degree of educational service (e.g., early intervention; home visits; school/education program for the deaf) from respective states due to hearing loss.

Demographics data were collected via a parent interview and background questionnaire. The interview was conducted with one or both parents of each child by one of the researchers. The interview gathered information regarding the child's level of hearing loss, school experience, and the presence of any cognitive or physical disabilities.

A background questionnaire was later completed by a parent of each child. The questionnaire gathered specific background information on the child (e.g., etiology, communication preference, etc.) and parent/household characteristics (e.g., level of education, home language, and family income).

Participating children (9 males; 11 females) ranged in age from 3:0 years to 5:7 with a mean age of 4:5 months. Table 1 provides an overview of the sample by age. The children's level of hearing loss ranged from mild to profound with one child having a unilateral loss. Five of the participating children had at least one cochlear implant, 12 used hearing aids, and three made use of no assistive listening device. Four children had recorded disabilities (not hearing loss), including dwarfism, CHARGE syndrome, Charcot-Marie-Tooth disease, and a physical malformation (cleft palate/missing pinna); parents reported at the time that these disabilities did not affect their children's level of cognitive functioning. A general search likewise indicated no effect on cognition due to the condition itself (Majors & Steltzer, 2008; Mayo Clinic Staff, 2012; National Institute of Neurological

**Table 1** Sample demographics by age

| | | Gender | | Hearing loss (better ear)[b] | | Assistive listening device | | | | Disability | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Age (years) | | Female | Male | Mild | Mod | Sev | Prof | Aid | CI | None | Yes | No |
| 3 | $X = 3.2$ ($n = 4$) | 2 | 2 | 1 | 2 | 0 | 1 | 4 | 0 | 0 | 1 | 3 |
| 4 | $X = 4.3$ ($n = 9$) | 5 | 4 | 0 | 1 | 3 | 4[a] | 4 | 3 | 2 | 1 | 8 |
| 5 | $X = 5.3$ ($n = 7$) | 4 | 3 | 0 | 2 | 2 | 3 | 4 | 2 | 1 | 2 | 5 |
| | | Parent hearing status | | Parent education (highest degree) | | | | | Income (in 1000) | | | |
| Age (years) | | D | H | HS | AS/ trade | BA | MA | PhD | <25 | 25–49 | 50–74 | ≥75 |
| 3 | $X = 3.2$ ($n = 4$) | 0 | 4 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 2 |
| 4 | $X = 4.3$ ($n = 9$) | 0 | 9 | 3 | 0 | 2 | 3 | 1 | 0 | 0 | 3 | 6 |
| 5 | $X = 5.3$ ($n = 7$) | 1 | 6 | 1 | 3 | 1 | 2 | 0 | 0 | 3 | 1 | 3 |

[a]one child with unilateral loss.
[b]one 4-year old with unknown loss.

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Disorders and Stroke, 2011; Richman & Eliason, 1982), although surrounding factors such as surgeries may have an indirect impact. For the majority of children, the primary means of communication at home was spoken English (Chinese was also spoken in one home and Spanish in another) with some of the families choosing to support the spoken language with the use of "sign language" (i.e., a variation of a signed English system such as Signing Exact English). One family used American Sign Language (ASL) predominantly. The sample includes a majority of families with reported yearly incomes of more than $75,000 (11) and highest educational degree to be bachelors or higher (11). There were no significant differences within the sample by age or gender, nor was there a significant difference in income by highest degree earned.

## Instrumentation

Two instruments used for data collection include the following.

*Test of Early Mathematics Ability (TEMA-3).* The TEMA-3 (Ginsburg & Baroody, 2003) is a standardized assessment designed to measure mathematics knowledge and skills associated with the concept of number in young children between 3.0 years and 8.0 years of age. The test is administered to each child individually. Results are available as raw scores, percentile ranks, age/grade equivalents, and descriptive scores (e.g., "average", "poor", "superior" etc.). The TEMA-3 was standardized based on the responses of 1,219 children whose characteristics mimic those reported in 2001 census information. Internal consistency reliabilities for the TEMA-3 are reported to be above .92 (Ginsburg & Baroody, 2003). The TEMA-3 has been used in studies with hearing children to assess young mathematical performance (Arnold, Fisher, Doctoroff, & Dobbs, 2002; Teisl, Mazzocco, & Myers, 2001), as well as in prior research to assess young d/hh children's mathematical performance (Kritzer, 2009), although it has not been normed on d/hh children. Testing was administered in the family home in one sitting for each participant.

Although the TEMA-3 provides useful information regarding young children's understanding of number-based concepts, an additional assessment tool was desired for the current study given its goals of 1) reporting on young children's understanding of mathematics in areas outside of number knowledge and skills; and 2) providing information beyond a correct/incorrect response examining the level of assistance that children needed in order to arrive at a correct response. Thus, a nonstandardized battery of performance-based tasks designed to examine young children's knowledge and skills in mathematics concepts in number and operations, geometry and spatial sense, measurement, and patterns, reasoning, and algebra was developed.

Performance Based Tasks (PBTs). Forty-six tasks with variation across three developmental levels—"low," "mid," and "high" (roughly paralleling ages 3, 4, and 5 years)—were designed to follow the trajectory of mathematics development in Appendix A. An example of the tasks (related to the concept of measurement) and their variations by developmental level is provided in Appendix B. (The complete set of tasks is available from the authors upon request.) The tasks were designed to appeal to preschool-aged children and thus involve hands-on manipulation of familiar, motivating, and engaging materials (e.g., cookies, brightly colored blocks, and dolls) as well as full body movement (e.g., jumping and moving about the room). For the concept of number, the following subconcepts were addressed: rote counting; counting objects; subitizing; cardinality; more/less; one-to-one correspondence; numeral recognition; numerical ordering; ordinal numbers; estimation; skip counting by 10; number sequencing; and numeral writing. Under geometry and spatial sense, tasks addressed part/whole relations; shape matching (identical, varying in size, varying in orientation), identification, sorting, and creation; and spatial orientation and direction. Tasks in measurement addressed time; size; length; weight; and volume. In problem solving and operations, tasks focused on story problems, composing and decomposing of numbers, number families and one to less correspondence. Finally, within patterns, reasoning, and algebra, subconcepts included matching, repeating and growing patterns, sequencing, sorting and categorization.

Each child began with tasks at his/her age level for each subconcept and proceeded up or down a level

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depending on his/her level of success with the initial task. The child was given three opportunities to be successful (i.e., respond with a correct answer) with most of the tasks—unassisted, guided, and modeled (some tasks did not allow for guidance and/or modeling). The first opportunity, "unassisted" presented the child with the stimulus only—no intervention by the administrator. If the child struggled with this, the administrator would provide the child with some guidance to the stimulus. If the child continued to struggle, the task was modeled for the child by the administrator, and the child was then expected to continue the task on his/her own. If the child was correct on a task unassisted, he/she was given the task at next higher level if available; if not, the child was given the next lower level task if available. The highest level of task at which the child was successful with any level of intervention was coded. The response was coded as "U" (unassisted) for each task answered correctly with no intervention; a code of "G" if he/she needed guidance; and a code of "M" if he/she needed modeling. The response was coded a "0" or incorrect if the child could not answer correctly regardless of the degree of assistance.

### Procedure

After receiving consent from parents and assent from each child, data collection ensued. Children were given the performance-based tasks first, followed by the TEMA-3. Children were tested at their homes; one child completed testing at his school. Each child took between one and two hours to complete all assessments in one or two sessions depending on attention span and availability. Multiple sessions took place within 48 hours of each other with the exception of one child whose second session took place one week after the first. The assessments were administered by one of the researchers in the child's preferred communication, while the other researcher attended to the cameras (2) (used to capture data for review, if needed, and dissemination), field coded, directed the tasks sequence (depending on the child's response), and noted any adverse occurrences such as the child not cooperating or an interruption. Any question on coding was noted, reviewed via video, and discussed until a decision was made.

## Results

### To what extent do young d/hh children demonstrate age appropriate understanding of early mathematics concepts in number (i.e., numbering, number comparisons, calculation, concepts, numeral literacy, number facts) as measured by the TEMA-3?

Table 2 shows the scores received on the TEMA-3 by all participating children in order of TEMA-3 rankings. As is shown, whereas at least one child scored in each rank category, the distribution of scores is heavy or slanted towards the lower abilities with 12 out of the 20 students scoring from 'below average' to 'very poor' and just 5 scoring 'above average' or better. When considered in terms of the age equivalent scores, five students scored six months or more above their chronological age (above average), five were within six months of their chronological age (average), and 10 were six months or more below their chronological age (below average).

### In what concept areas (i.e., number and operations; geometry and spatial sense; measurement; patterns, reasoning, and algebra) do young d/hh children show strength and weakness?

Tables 3–7 describe the children's performance on each of the 46 tasks. Each task, presented in the left-hand column, is divided into three levels—low, mid, and high— approximating typical skill level at ages 3, 4, and 5 years, respectively. Each task level is further split into three columns to describe the degree of intervention needed in order for the child to be successful on the task. To reiterate, a code of "U" means that the child was able to complete the task unassisted, a code of "G" means that the child needed guidance, a code of "M" means that the child needed a model in order to

**Table 2** Frequency of TEMA-3 scores and rankings

| TEMA-3 ability score | TEMA-3 rank | Number of participants ($n$ = 20) |
|---|---|---|
| >131 | Very superior | 1 |
| 121–130 | Superior | 2 |
| 111–120 | Above average | 2 |
| 90–110 | Average | 3 |
| 80–89 | Below average | 5 |
| 70–79 | Poor | 3 |
| <69 | Very poor | 4 |

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**Table 3** Frequency of response in number tasks by age

| Number | Age (years) | 0 | Low M | Low G | Low U | Mid M | Mid G | Mid U | High M | High G | High U |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Counting (rote)** | 3 | 3 | | | | | | 1 | | | |
| | 4 | 5 | | | 1 | | | 2 | | | 1 |
| | 5 | 1 | | | | | | 1 | | | 5 |
| Counting (objects) | 3 | 2 | | | 2 | | | | | | |
| | 4 | 2 | | 1 | 2 | | 1 | 2 | | 1 | |
| | 5 | 1 | | 1 | | | | | | 1 | 4 |
| **Counting (subitizing—stating number of elements without counting)** | 3 | 4 | | | | | | | | | |
| | 4 | 8 | | | | | | | | 1 | |
| | 5 | 1 | | | | | | 1 | | 5 | |
| Counting (cardinality) | 3 | | | | | | | | | | |
| | 4 | 7 | | | | | | 2 | | | |
| | 5 | 1 | | | | | 1 | 5 | | | |
| **Counting (more/less—determining which of two sets/numerals is more)** | 3 | 3 | | | 1 | | | | | | |
| | 4 | 8 | | | | | | 1 | | | |
| | 5 | 1 | | | | | | 2 | | 1 | 3 |
| **One-to-one Correspondence** | 3 | 2 | 1 | | 1 | | | | | | |
| | 4 | 4 | 1 | 1 | 1 | | | 2 | | | |
| | 5 | 1 | | | | | | 6 | | | |
| **Numeral recognition (naming)** | 3 | | | | | | | 1 | | | |
| | 4 | 7 | | | | | | 2 | | | |
| | 5 | 2 | | | | | | 5 | | | |
| **Numeral recognition (order)** | 3 | | | | | | | | | | |
| | 4 | 7 | | | | | | 2 | | | |
| | 5 | 1 | | | | | | 6 | | | |
| **Ordinal** | 3 | | | | | | | | | | |
| | 4 | | | | | | | | | | |
| | 5 | 5 | | | | | | | | | 2 |
| **Estimation** | 3 | 4 | | | | | | | | | |
| | 4 | 9 | | | | | | | | | |
| | 5 | 5 | | | | | | 1 | | | 1 |
| **Counting (skip by 10s)** | 3 | | | | | | | | | | |
| | 4 | | | | | | | | | | |
| | 5 | 3 | | | | | | | 4 | | |
| **Number sequence (before/after)** | 3 | | | | | | | | | | |
| | 4 | 9 | | | | | | | | | |
| | 5 | 4 | | | | | | | 1 | 1 | 1 |
| **Numeral writing (4, 8, 6)** | 3 | | | | | | | | | | |
| | 4 | 9 | | | | | | | | | |
| | 5 | 1 | | | | | | 1 | | | 5 |

be successful on the task. A score of "0" means that the child was unable to complete the task regardless of the degree of assistance offered. Shaded areas indicate that no task was available at that level (therefore the *n* for these tasks will be less than 20). All three ages of children are represented on the table. Thus, the table shows results within each subconcept for which children performed at, above, or below their expected age level, given a measure of assistance. Further, the table indicates in bold lettering those subconcepts for which 50% or more of the children were unsuccessful in a task appropriate for their age. For example, Table 3 shows the subconcept "Counting by Rote" as bolded. Nine children (three 3-year olds, five 4-year olds, and one 5-year old) out of 20 were unable to complete even the low level of this task successfully; that is, they could not count to five from memory. In addition, the table shows that one 4-year old completed the

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**Table 4** Frequency of response in geometry tasks by age

| Geometry | Age (years) | 0 | Low M | Low G | Low U | Mid M | Mid G | Mid U | High M | High G | High U |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Puzzles** | 3 | 2 | | 1 | 1 | | | | | | |
| | 4 | 5 | 1 | | 2 | | 1 | | | | |
| | 5 | 1 | | | 2 | 1 | 2 | 1 | | | |
| Shape (matching identical) | 3 | | | | 4 | | | | | | |
| | 4 | | | | 9 | | | | | | |
| | 5 | 1 | | | 6 | | | | | | |
| Shape (matching vary in size) | 3 | 1 | | | 3 | | | | | | |
| | 4 | 1 | | | 8 | | | | | | |
| | 5 | 1 | | | 6 | | | | | | |
| **Shape (matching vary in orientation)** | 3 | 2 | | | 2 | | | | | | |
| | 4 | 5 | | | 4 | | | | | | |
| | 5 | 3 | | | 4 | | | | | | |
| **Shape (sorting)** | 3 | 4 | | | | | | | | | |
| | 4 | 9 | | | | | | | | | |
| | 5 | 2 | | | 1 | | | | | 3 | 1 |
| Shape (naming) | 3 | 1 | | | | | | 1 | | | 2 |
| | 4 | 5 | | | 1 | | | 1 | | | 2 |
| | 5 | 2 | | | | | | | | | 5 |
| **Shape (creating)** | 3 | | | | | | | | | | |
| | 4 | 8 | | | | | | 1 | | | |
| | 5 | 4 | | | | | | | | | 3 |
| Spatial orientation (left/right) | 3 | | | | | | | | | | 1 |
| | 4 | | | | | | | | | | |
| | 5 | 3 | | | | | | | | | 4 |
| Spatial orientation (on, under, etc.) | 3 | | | | | | | | | | |
| | 4 | | | | | | | | | | 1 |
| | 5 | 3 | | | | | | | | | 4 |

task (unassisted) at the low level and one 5-year old completed the task (unassisted) at the mid level, each a level below their respective ages. There was one 3-year old and one 4-year old, however, who completed the task a level above their respective ages. We highlight results by concept in the next several paragraphs.

Table 3 shows results for the concept of number. Well over half (8 out of 13) of the tasks in number were either not solved successfully by at least half of the children for whom the task was appropriate or solved at a level below his/her age. The weakest area was estimation as just one child was successful with the task at his/her age level. Subconcepts of ordinal numbers, distinguishing more/less, and numeral writing were very weak as well with an approximate 30% success rate. Areas of strength, or tasks in which more than 50% of the children were successful at their age level or above, albeit with some degree of assistance, included object counting and skip counting by 10s.

Geometry (Table 4) was the strongest concept area for the children with just four of the nine subconcepts accomplished by less than 50%. Children showed particular strength in matching shapes that were identical or differing in size only. Children, including five out of nine 4-year olds and three out of seven 5-year olds, had difficulty, however, in matching shapes that varied in orientation. Of particular difficulty in geometry was sorting of like shapes into groups. Only four children accomplished the task at their age level, three of which with guidance. Another weak subconcept in geometry was creating shapes from memory. Most children were unable to create a shape other than a circle (i.e., square, triangle, or rectangle). Puzzles at the various levels, which mathematically incorporate both matching, orientation, and part to whole, also proved to be a difficult task for the children. Only three children, and no 5-year olds, could complete an appropriate age level puzzle and only one of those children unassisted.

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**Table 5** Frequency of response in measurement tasks by age

| Measurement | Age (years) | 0 | Low | | | Mid | | | High | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | M | G | U | M | G | U | M | G | U |
| **Time (day/night; days; months)** | 3 | 2 | | 1 | 1 | | | | | | |
| | 4 | 5 | | 1 | 2 | | | 1 | | | |
| | 5 | 2 | | | 1 | | | 3 | | | 1 |
| **Time (seasons)** | 3 | | | | | | | | | | |
| | 4 | | | | | | | | | | 1 |
| | 5 | 4 | | | | | | | | | 3 |
| **Time (before/after; more/less)** | 3 | | | | | | | 1 | | | |
| | 4 | 7 | | | | 1 | | | | | 1 |
| | 5 | 3 | | | | | 1 | 1 | | | 2 |
| Size (vocabulary) | 3 | 1 | 1 | | 2 | | | | | | |
| | 4 | 3 | 3 | | 3 | | | | | | |
| | 5 | 1 | | | 6 | | | | | | |
| **Length (order pictures)** | 3 | | | | | | | | | | |
| | 4 | 7 | | | | | | 2 | | | |
| | 5 | 4 | | | | 1 | | 2 | | | |
| **Length (order objects)** | 3 | | | | | | | | | | |
| | 4 | 8 | | | | | | 1 | | | |
| | 5 | 2 | | | | | | 1 | | | 4 |
| **Length (ident. of missing)** | 3 | | | | | | | | | | |
| | 4 | | | | | | | | | | |
| | 5 | 6 | | | | | | | | | 1 |
| **Length (nonstandardized Measurement)** | 3 | | | | | | | | | | |
| | 4 | | | | | | | | | | |
| | 5 | 5 | | | | | | | | | 2 |
| **Weight (objects, pictures, scale)** | 3 | 1 | | | 2 | | | 1 | | | |
| | 4 | 4 | 2 | | 3 | | | | | | |
| | 5 | 1 | | | 1 | | | 1 | | | 4 |
| **Volume** | 3 | | | | | | | | | | |
| | 4 | 6 | | | | | | 3 | | | |
| | 5 | 3 | | | | | | 4 | | | |

Children seemed to have great difficulty with the majority of measurement subconcepts (9 out of 10 bolded in Table 5). Only the subconcept of relative size, at the most basic level—that of general vocabulary (big, small), was accomplished successfully by more than 50% of the children. Although this was a strength, eleven out of sixteen children, including four 5-year olds, could not order a series of four pictured animals from smallest to biggest (a mid task). Other challenging areas included time where nine children, including five 4-year olds and two 5-year olds could not sort pictures into events that happen during the day and those that happen at night. Only one child, a 5-year old, could name months of the year subconcept. In addition, 10 out of the sixteen 4- and 5-year olds could not choose the correct picture that came either before or after an event (i.e., stating which of two pictures came before/after a given picture).

Although the majority of subconcepts were at an overall higher level of difficulty, problem solving/operations was the weakest concept area with few children solving any of the tasks at their age level or at all. In story problems, just seven out of the 20 children could solve the following low level task with the correct number of manipulatives provided them: "You have two cookies (given). I am going to give you three more (given). How many do you have now?" In addition, 12 out of 16 children (4- and 5-year olds) could not indicate how many more cookies would be needed so that there would be five on a frame, nor could take away the correct number of cookies from five given on a frame to leave the target amount. Finally, just two 4-year olds and three 5-year olds could determine how many more cookies were needed when asked to give each of 20 toy girls a cookie – a four year old task on the developmental trajectory.

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**Table 6** Frequency of response in problem solving tasks by age

| Problem solving | Age (years) | 0 | Low | | | Mid | | | High | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | M | G | U | M | G | U | M | G | U |
| **Story problems** | 3 | 3 | | | 1 | | | | | | |
| | 4 | 8 | | | 1 | | | | | | |
| | 5 | 2 | | 1 | 4 | | | | | | |
| **5-frame plus (add to make 5)** | 3 | | | | | | | | | | |
| | 4 | 9 | | | | | | | | | |
| | 5 | 3 | | | | | | 4 | | | |
| **5-frame minus (remove from 5)** | 3 | | | | | | | | | | |
| | 4 | 8 | | | | | | 1 | | | |
| | 5 | 4 | | | | 1 | 1 | 1 | | | |
| **5-frame combo (decomposing 5)** | 3 | | | | | | | | | | |
| | 4 | | | | | | | | | | |
| | 5 | 6 | | | | | | | | | 1 |
| **10-frame combo (decomposing 10)** | 3 | | | | | | | | | | |
| | 4 | | | | | | | | | | |
| | 5 | 6 | | | | | | | | | 1 |
| **One to one less (how many more needed to complete 1:1)** | 3 | | | | | | | | | | |
| | 4 | 7 | | | | | 1 | 1 | | | |
| | 5 | 4 | | | | 1 | | 2 | | | |

**Table 7** Frequency of response in pattern tasks by age

| Patterns, reasoning, algebra | Age (years) | 0 | Low | | | Mid | | | High | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | M | G | U | M | G | U | M | G | U |
| Matching | 3 | 2 | | 1 | 1 | | | | | | |
| | 4 | 1 | | 1 | 7 | | | | | | |
| | 5 | | | | 7 | | | | | | |
| **Pattern—simple repeating (ABAB)** | 3 | 3 | 1 | | | | | | | | |
| | 4 | 7 | | 1 | | | | | | | 1 |
| | 5 | 1 | | | | | | 2 | | | 4 |
| **Pattern—growing (A, AB, ABC)** | 3 | | | | | | | | | | |
| | 4 | | | | | | | | | | |
| | 5 | 6 | | | | | | | | | 1 |
| **Sequencing (first, next, then, last)** | 3 | 4 | | | | | | | | | |
| | 4 | 8 | | | | | | 1 | | | |
| | 5 | 2 | | | 2 | | | 3 | | | |
| **Sorting (color)** | 3 | | | | | | | | | | |
| | 4 | 5 | | | | | 1 | 3 | | | |
| | 5 | 3 | | | | | | 4 | | | |
| **Sorting (function)** | 3 | | | | | | | | | | |
| | 4 | 6 | | | | | 1 | 2 | | | |
| | 5 | 3 | | | | 2 | | 2 | | | |
| Sorting (two characteristics) | 3 | | | | | | | | | | |
| | 4 | | | | | | | | | 1 | 1 |
| | 5 | 2 | | | | | | | 1 | 1 | 3 |
| **Sorting (Venn)** | 3 | | | | | | | | | | |
| | 4 | | | | | | | | | | |
| | 5 | 6 | | | | | | | | 1 | |

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Finally, as Table 7 shows, children also had difficulty in subconcepts related to patterns, reasoning, and algebra. Children were especially weak in extending repeating patterns (with 11 children unsuccessful in extending an ABAB color pattern) and sequencing events (e.g., first, next, then, last). Areas of relative strength included matching (i.e., pairing like socks) and sorting by two characteristics (e.g., red and square).

## Discussion

As discussed earlier, the field of deaf education and specifically those professionals concerned with the mathematics performance of d/hh students has been left to speculate as to the origin of the "math gap" and its possible implications for academic success in all areas. The results from the present study show d/hh students' poor achievement in mathematics is indeed present in the preschool years. The TEMA-3 data indicated that half of the children tested below average giving indication even at this early age of weaknesses in their understanding of foundational mathematics concepts in number and problem solving. Although slightly more optimistic, these score rankings are similar to those found in Kritzer (2009) in which all participating children (with the exception of an outlier) scored at the level of "average" or below. One possible explanation for this difference is the inclusion of 3-year-old children in the current study. These children tended to receive the lowest scores possible on the test; however, due to their age and the test's scoring system, they were included in the category of "average," inflating this group. In addition, 6-year old children were included in the Kritzer (2009) study. Because the "math gap" seems to widen with age, the mean performance score in the Kritzer study was more depressed with older children participating.

The present study further provides previously missing information regarding d/hh children's performance with specific concept areas in early mathematics outside of number. The findings here not only show areas of strength (geometry) and areas of weakness in mathematics development (problem solving and measurement) for d/hh children but also indicate the level of their understanding in other areas, including patterns, reasoning and algebra, as well as number, and in subconcepts within all areas. Such information can provide more specific indicators of potential breakdowns in the knowledge and use of foundational mathematics concepts and skills. Given that understanding of early mathematics concepts especially in number has been linked to later academic achievement, this information can have significant implications for d/hh children's academic success, perhaps providing a "map" of sorts by which professionals and parents can guide learning.

In addition, the current study's findings suggest that young children with hearing loss may not have the foundational skills they need to address mathematics concepts such as those listed within the Common Core Standards (http://www.corestandards.org/) (adopted by 45 states and 3 U.S. territories) upon entering kindergarten. For example, according to the Common Core Standards, young children during kindergarten should develop the ability to count upto 100 by 1s and 10s. While there is some debate as to the validity/basis NW of the Common Core Standards (Given that in the current study only seven out of 20 children demonstrated an ability to count to a number appropriate for their age and nine children (including one 5-year-old) were unable to count by ones to five, a delay in this area may be likely. These results support beliefs that d/hh children are approximately two years behind in counting. This delay may cause interruptions in other areas in which mathematics knowledge and skill is needed as well. For example, a delay in knowledge of the counting string past five would certainly deter the child's ability to decompose numbers up to ten into different pairs, another Common Core Standard for the kindergarten year. Further, results from the current study showed that the majority of children (7 out of 8 for whom the task was appropriate) were unable to decompose the number five into different pairs even with manipulatives and intervention. The Common Core Standards also sets concepts and skills in measurement and geometry for kindergarten-age children, which again according to the current data, may be difficult for d/hh children without attention at the preschool level.

Although there were weak subconcepts within each area by age group, it is most important to note those subconcepts which were weak *across* age groups, as they

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may indicate a more serious shortfall. These include within number, the subconcept of estimation to which 18 out of 20 children in the study could not respond correctly. Estimation is a critical part of number knowledge and later learning of fluency with mathematics facts, as well as a significant part of all areas including problem solving fractions, and measurement. It is also necessary for the large majority of school-based curricula (Bana & Dolma, 2004; Doman, 2009; NCTM, 2000).

Time and sequencing were other areas where at least half of the children were below their age/development level. These subconcepts in measurement are critical elements in problem solving, giving story problems their structure, as well as in other disciplines such as reading. For example, in the following story problem, "John had some candies. He ate 3 of them. Now John has 2 candies left. How many candies did John have before," a child would need to understand the chronological sequence of events and the change over time in order to solve the problem. In addition, a weakness in understanding of time (such as knowing the days of the week) and time-related vocabulary (such as "daily" or "monthly") may further hinder problem solving and likewise have an effect on reading comprehension. Whereas it is understood that concepts of time are abstract and develop later on, they can be represented to young children concretely or pictorially. In addition, development of time concepts may also be addressed through frequent, intentional linguistic exposure, including the use of related vocabulary (e.g., nighttime, daytime, morning, afternoon, etc), sequencing (e.g., first we will… next…, etc.), and use of specific language related to time (e.g., "We will leave the playground in 5 minutes.").

Although the results here do not answer why d/hh students have such difficulty with mathematics concepts, a strong possibility is the fact that mathematics development may be hindered by absent, inappropriate, or misguided learning opportunities. A d/hh child's limited experiences and reduced ability to access incidental learning because of language barriers and/or the possible lack of understanding that adults (parents and early interventionists) have regarding the development of mathematics concepts, as well as a pervasive belief that mathematics is 'not as important' as language and/or literacy may contribute to poor academic performance in this area (Gregory, 1998; Nunes, 2004; Pagliaro, 2006). Support for this hypothesis comes from two studies. In one study by Pagliaro and Kritzer (2010), it was found that parents who incorporated "learning behaviors" (i.e., mediation behaviors such as focusing attention, asking questions, linking the present to past and future events, and providing specific praise) into their natural interactions with their children had children who were better able to engage with their learning environments and who showed relatively higher mathematics ability than those who did not. Essentially, these children knew how to learn through the parent modeling and mediation. Another study which investigated the initial phase of the MRPP intervention program also showed promising results from parent mediation of their d/hh children's early mathematics learning. In this study (Kritzer & Pagliaro, 2012), participation in the intervention which was based on increasing the mathematics learning opportunities of preschool d/hh children resulted in an overall increase in the frequency and variety of mathematics concepts parents discussed with their children, thus exposing them to various subconcepts naturally in their lives.

## Limitations

Findings reported here should be accepted judiciously due to the existence of several limitations, including the following. First, the instruments used to collect the data each present their own shortcomings. Although it has been used in previous research with d/hh children, the TEMA-3 has not been normed with this population, and the performance-based tasks used in this research, developed by the researchers, have not been formally tested for reliability and validity with either hearing or d/hh children. Although we are confident in the data that have been collected, these issues in assessment should be considered.

A second limitation to the present study was the small number of participants from a specific geographical location. Whereas twenty children in this age group is relatively substantial within deaf education research, the distribution across various categories did not allow for statistical analyses to determine significant differences. In addition, our sample consisted of more

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families from a higher SES, and of children who are primarily oral in their communication preferences. It is unclear if or how these two factors may have affected the results. Given the positive correlation between SES and achievement (Sirin, 2005), the 'gap' found here may be better than what would be expected. Future studies with larger Ns, normally distributed and from across the U.S. and internationally are desired.

Third, working with young deaf children brings its own set of challenges, including limited attention span, "stranger anxiety," and language/communication preferences and disabilities that may have not yet been determined or identified. All of these factors may have influenced the results of the current study.

Finally, the trajectory used to guide the study including the design of its PBTs was based on typical, hearing children. The field has yet to establish how d/hh children learn and organize concepts given their unique experiences and circumstances. It may be, therefore, that d/hh children follow a different path of development. More studies, particularly longitudinal investigations, are needed to determine if d/hh children are truly behind hearing children or if they simply follow a different path.

Notwithstanding the limitations of the study, the results present strong evidence that d/hh students' difficulties in mathematics begin prior to the start of formal schooling and that these weak foundational skills in various areas of mathematics may be the root of the poor achievement in computation and problem solving experienced by d/hh students for more than four decades. It is critical then that early intervention and early childhood educators of d/hh students work with parents to establish a robust and comprehensive foundation for future mathematics learning in their children on which more advanced mathematics knowledge, required for a full and contributing participation in tomorrow's society, can be built.

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## Conflicts of Interest

No conflicts of interest were reported.

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**Appendix A**

| Mean Age | Number | Geometry & Spatial Sense | Measurement | Operations & Problem Solving | Patterns, Reasoning, & Algebra |
|---|---|---|---|---|---|
| 2 years | • Understands and distinguishes 1 and 2 from many;<br>• Rote counts to at least 3 (may be incorrect beyond 3) | • Orders by size<br>• Completes simple 3–4 piece puzzles | • Explores objects by filling and emptying containers<br>• Begin to understand and describe same and different | • Can nonverbally determine that one item added to another makes "two" | • Learning order of day<br>• Begin to notice patterns<br>• Begin to use terms for "today", "tomorrow" "yesterday"<br>• Show interest in pattern or sequence (e.g., stringing beads)<br>• Begins to logically sort and classify |
| 3 years | • Understands and distinguishes 1, 2, & 3 from many;<br>• Rote count to 5, possibly 10;<br>• Counts objects to 5;<br>• Demonstrates one-to-one correspondence up to 3;<br>• Uses "more" to identify larger of two collections (if difference is obvious) | • Completes 4–10 piece puzzle<br>• Understands that wholes can be separated into parts (e.g., a pizza)<br>• Match shapes with same size and orientation<br>• Experiments with spatial labels, may not be correct<br>• Identifies basic shapes (circle, square, possibly triangle) | • Develop sense of time for regular events (knows basic sequence of day)<br>• Develops early time concepts (morning, afternoon, night, later, soon)- sometimes confuses yesterday, today, and tomorrow<br>• Recognizes differences between measurable concepts | • Using context and objects, begin to nonverbally solve problems involving sums up to "five";<br>• Intuitively recognize that if you change the size of part of a collection you also change the whole | • Can identify repeating unity and begin to extend a simple pattern<br>• Sort arbitrarily ("because I like it")<br>• Match perceptually by characteristics that are observable (e.g., match socks that are the same) |

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**Appendix**
Continued

| Mean Age | Number | Geometry & Spatial Sense | Measurement | Operations & Problem Solving | Patterns, Reasoning, & Algebra |
|---|---|---|---|---|---|
| 4 years<br>Pre-K | • Rote count to 10 (correctly);<br>• Counts 5–10 objects;<br>• Subitizes to 4;<br>• Recognizes that last number counted represents the total;<br>• Understands that numbers later in counting sequence are larger (up to 10)<br>• Recognizes/reads numerals 0–9 | • Completes puzzle with up to 15 pieces<br>• Match shapes with different size and orientation<br>• Recognize and name simple shapes and some variations (circle, square, triangle, rectangle)- does not yet make a distinction between sides and corners;<br>• Build, copy and describe 2 dimensional shapes<br>• Begin to use/understand words representing physical relations or positions (over, under, left, right, etc.) in particular- those that remain regardless of perspective<br>• Uses simple map to find a hidden object | • Figure out that different size containers hold more or less<br>• Develop language to describe attributes (big, small, long, tall, short, heavy, light, fast, slow)<br>• Can describe how things are "same" or "different"<br>• Can compare a single attribute across several objects (e.g., sizes of pieces of cake)<br>• Can order objects from smallest to largest and describe the relationship among them<br>• Can recite days of week<br>• Knows time associated with specific events<br>• Can compare length or area of two specific objects using direct comparison | • Using objects, can mentally determine sums up to "five" and their subtraction counterparts | • Understand a sequence of events when explained (e.g., first, next, last)<br>• Can complete an adult-imposed classification task- sticking with one observable feature (e.g., sort by color, shape, size) |

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**Appendix**
Continued

| Mean Age | Number | Geometry & Spatial Sense | Measurement | Operations & Problem Solving | Patterns, Reasoning, & Algebra |
|---|---|---|---|---|---|
| 5 years K | • Rote count to 20;<br>• Counts 10–20 items;<br>• Subitizes up to 5;<br>• Names number before and after up to 30;<br>• Counts backwards from 10;<br>• Skip counts to 100 by 10s<br>• Makes reasonable estimates up to 20; Understands terms related to estimation (about, closer to, between, etc.);<br>• Understands that numbers later in counting sequence are larger (up to 100);<br>• Can use mental number line to determine relative proximity up to 9;<br>• Can copy or write numerals 0–9 and connect them to quantitative value | • Continues to recognize simple shapes that vary in size and orientation (more complex variations than previous level)<br>• Copies shape from memory after seeing model briefly<br>• Matches shapes with objects that have same shape or size (congruence)<br>• Identifies more complex shapes by name (e.g., hexagon) and distinguishes sides from corners;<br>• Completes increasingly complex puzzles (smaller pieces, more numerous)<br>• Understands and uses words to represent physical relations/ positions, including spatial labels that change based on perspective (e.g., left, right)<br>• Creates a simple map | • Strong sense of time- know when familiar events take place<br>• Know days, months, and seasons<br>• Beginning to learn to tell time<br>• Begin to understand conservation (number remains the same unless items are added or removed)<br>• Compare length of two objects by representing length with one unit (e.g. string)- then comparing two strings<br>• May begin to measure length of an object using same-size units laid end-to-end | • Can nonverbally and mentally determine sums up to “five” and their subtraction counterparts (labels all arrangements up to 5, then 10 – e.g., 1&4, 2&3)<br>• Can use informal knowledge to estimate the sums of addition word problems<br>• Begin to use concrete counting strategies to solve addition story problems<br>• Can use various addition strategies to mentally determine sums up to “nine”<br>• Understands “part-whole” relationship of addition; can solve part-part-whole story problem with missing whole and sums up to “10”<br>• Can group objects into 5’s or 10’s<br>• Recognizes that position of a digit in a number affects its value<br>• *Can read multidigit numerals up to “19”<br>• *Can use informal strategies to solve fair-sharing problems | • Can sort and classify by more than one feature and articulate why items are grouped the way they are (features may not be directly observable- e.g., function, number)<br>• Begin to understand growing pattern involved with counting (“one” is added to get to the next number)<br>• Begins to use letters to represent the core of a repeating pattern<br>• Can use deductive reasoning to solve a simple problem (e.g., figures out who is missing by looking at children present)<br>• Can sequence events chronologically<br>• Begin to construct algebra sense- can use problem solving strategies to solve a problem (e.g., draw a picture, draw and adjust, work backwards) |

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**Appendix**
Continued

| Mean Age | Number | Geometry & Spatial Sense | Measurement | Operations & Problem Solving | Patterns, Reasoning, & Algebra |
|---|---|---|---|---|---|
| 6 years<br>1st gr. | • Can count to "200" by rote;<br>• Counts 20 items;<br>• Can name number before/after up to 99;<br>• Counts backwards from 20;<br>• Beginning to count by 5's to 100;<br>• Beginning to count by 2s to 20;<br>• Can make reasonable estimates up to 100;<br>• Can use mental number line to determine relative proximity with two-digit numbers;<br>• Can identify written number words 1–9 and use them to represent number of items in a collection;<br>• Can describe parallels between ordinal terms (e.g., 1st, 2nd, 3rd) and cardinal terms | • Identifies and counts sides of shapes; beginning to count angles<br>• Can use class names to classify and sort shapes<br>• Can accurately visualize two-dimensional shapes and draw them from memory<br>• Can define congruence and match shapes to show congruence<br>• Can create new shapes by combining shapes<br>• Can give and follow directions for moving in physical space and on a map | • Understands that unless more is added or removed, the number of objects in a collection remains the same even if the appearance changes<br>• Can measure using same size unit laid end to end<br>• Can make informal comparisons and estimates | • Begin to count-on to solve a problem (e.g., to solve 3+2, counts on from 3)<br>• Labels arrangements up to 20 (e.g., 4 groups of 5)<br>• Can break a larger unit (e.g., 100) into smaller units and combine smaller units to make larger units<br>• Can read multi-digit numerals up to 99<br>• Can accurately write multi-digit numbers up to 999<br>• Can solve fair sharing problems where up to 20 items are divided among 3 to 5 people<br>• Begins to compare simple fractions (1/2, 1/4,. 1/3)<br>• Can solve repeated addition problems using objects, mentally, or through use of verbal counting | • Understands even and odd numbers<br>• Begin to understand that the "equals" sign can be interpreted as "the same as"<br>• Begin to understand the "other name for a number" concept (e.g., 12 = 12 + 0; 11 + 1, etc.)<br>• Can sort and classify on the basis of more than one characteristic and/or consider an implicit relationship |

Mathematics Development Grid adapted from: http://www.pbs.org/parents/childdevelopmenttracker.html NCTM Standards, 2000; Sarama & Clements, 2009.

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**Appendix B**

| Measurement | Low | Medium | High | Guide | Model |
|---|---|---|---|---|---|
| Codes | U | | | G | M |
| Time | DAY/NIGHT: Show child Day/Night board and 6 activity pictures. Say, "Put the day activities with the sun and the night activities with the moon." | DAYS: Show child calendar. Focus on one week- Point to Sunday, ask, "What's next?" until all 7 days are named. | MONTH: Show child a picture of a turkey. Ask when we see a lot of turkeys (Thanksgiving). Ask what month Thanksgiving is in. Repeat above with 3 holiday pictures (Halloween, Independence Day, Christmas tree/menorah). Ask child to explain the month associated with each holiday. | DAY/NIGHT: Say, "Look at this picture, the mommy and boy are reading a book. It looks like the boy is in bed. Where should we put this picture?". | DAY/NIGHT: Say, "I brush my teeth at night before I go to bed. I'm going to put the toothbrushing picture with the moon. What else do we do at night?" |
| | | | | DAYS: Ask "What is today?" "Tomorrow is what?" | DAYS: While pointing to the calendar say, "Today is [Monday]. Tomorrow is [Tuesday]. What's next?" |
| | | | | MONTH: Focus child's attention on the picture. Talk about season related elements. Ask, "what time of year do you think it is in this picture?" Ask child what month it might be | MONTH: While pointing to the picture. Say, "This is a turkey. We see them at Thanksgiving. Thanksgiving is in the month of November." Point to Jack-o-Lantern and continue the task. |
| Time | | BEFORE/AFTER: Show child 2 sequential prompt pictures plus one 'before' picture and one 'after' picture separated. Ask child to explain what is happening in the prompt pictures. Ask child to pick the appropriate before/after picture for the event, as appropriate. | MORE TIME: Show child, "How much time" pictures (2). Ask child to explain what is happening in the picture. Ask child to point to picture that takes more time. | BEFORE/AFTER: Tell story of prompt pictures. Ask child to label bottom pictures. Ask, "What happened after/before- [story]?" | BEFORE/AFTER: first picture of "Before" and first picture of "after"?" MORE TIME: first picture, have child do second independently. |

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**Appendix**
Continued

| Measurement | Low | Medium | High | Guide | Model |
|---|---|---|---|---|---|
| Codes | U | | | G | M |
| | | | | MORE TIME: Tell story of prompt pictures. Ask, "Which takes" more time? Which one takes a long time?" | |
| | | | SEASONS: Show child 4 pictures- one per season- ask child to talk about the picture. Ask child to explain the picture, watch for language related to season. | | SEASONS: Say, "This picture shows winter. See the snow?" Have child name season in remaining pictures. |
| Relative Size Vocabulary | BEARS, PENCIL, TREES: Show child picture. Ask child to describe what they see. Prompt if needed by asking if the items are the same or different? Different how?" | | | BEARS, PENCIL, TREES: Say, "These two bears are different. Look at the size. How are they different?" | BEARS, PENCIL, TREES: Say, one [bear] is big, one [bear] is small. Which one is big? Continue task with pencils picture. |
| Relative Size Order | | ANIMAL PICTURES: Give child 4 pictures of different sized animals (mouse, cat, cow, elephant) in random order. Ask child to name the animals. Ask child to put the pictures in order from smallest animal to biggest. | | ANIMAL PICTURES: Name the pictures. Ask, "Which is the smallest?" Child orders the rest. OR Compare two pictures close to each other in size, ask, "Which one is bigger?" | ANIMAL PICTURES: Place smallest picture first. Tell child, "This mouse is the smallest so I will place it here. What animal would be next?" Child orders rest. |
| Length: Ordered, continuous unit objects | | STRAWS: Give child three straws of same color, different sizes. Say, "Put these straws in order from smallest to biggest." | STRAWS: Repeat with 4 sticks; 5 sticks | STRAWS: Say, "Which one is smallest? Next?" | STRAWS: Line sticks up along an edge for the child starting with the smallest. Ask which would be next. |

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**Appendix**
Continued

| Measurement | Low | Medium | High | Guide | Model |
|---|---|---|---|---|---|
| Codes | U | | | G | M |
| Length: Ordered, discrete unit objects | | | TOWERS: Give child 6 Unifix block towers. Say, "Put these in order from smallest to biggest." (Part 2) Out of child's view, remove the third tower. Ask, "Which one is missing? How do you know?" | TOWERS: Say, "Which tower is first/smallest? Next?" (Part 2) Say, "Remember, it looked like stairs before." | TOWERS: Line first tower up along an edge, line second- say. "I'm putting this one next because it is a little bit bigger. Do the rest?" (Part 2): Count the cubes in the first two towers. |
| Length: Non-standardized units | | | JUMP: Say to child, "I can jump as far as 4 popsicle sticks! Do you think you can beat me?" Have child jump. Mark end point. Give child sticks and ask him/her to see if his/her jump beat "my jump." (Looking for end to end measurement from start to end.) | JUMP: Say, "What do you think you should do with those sticks? How can you use them to measure your jump?" | JUMP: Lay out first two sticks end to end. Have child continue. |
| Weight | BOXES: Give child a set of 2 differently weighted boxes. Say, "Pick them up. Which one is heavier? | PICTURES: Show child pictures of a brick and a feather. Ask which is heavier. Continue with two more sets of pictures. | SCALE: Give child a small kitchen scale and 4 bags of sugar packets of differing weights. Say, "Which is heavier? Use the scale to figure it out." | BOXES: Say, "Weigh both boxes in your hands (like a scale). Which one is heavier – this one or this one?"<br>PICTURES: Ask the child to name the pictures and relate them to their experience. Ask which they know to be heavy. | BOXES: Weigh two boxes and explain that "this one" is heavier because of its weight. Have child do the next two sets of boxes on own.<br>PICTURES: Name the pictures and discuss how it would be harder to carry the brick because it is heavy. Continue with next two sets of pictures. |

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**Appendix**
Continued

| Measurement | Low | Medium | High | Guide | Model |
|---|---|---|---|---|---|
| Codes | U | | | G | M |
| | | | | SCALE: Say, "What will happen when you put each bag on the scale? What does it mean when the line moves?" Ask the child to find the heavier bag. | SCALE: Say, "When I put this bag on the scale, the line went to a (number). What happens when I put this bag on?" Ask child to weigh the rest of the bags and find which is heaviest?" |
| | | BEADS: Show child two containers and a bag of red beads. Say, "If I want to put all of these beads into a container, which container should I use? This one or this one? Why? | | BEADS: Repeat question with exaggerated pouring gesture – say, "I don't want any beads left over. Which container is better?" | |

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